Heat Transfer Between Counterflowing Fluids Separated by a Heat-Conducting Plate

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The heat transfer between two counterflowing fluids separated by a heat-conducting plate was analyzed numerically. In a conventional analysis, it is assumed that a one-dimensional heat flow transverse to the plate prevails, i.e., the axial conduction along the plate is neglected. The present results consist of a comparison between cases where the axial plate conduction was ignored to those where it was included. Axial fluid conduction was always included in the analysis. The heat-transfer performance and the distributions of bulk fluid and plate temperatures are presented for various Peclet numbers and values of a dimensionless wall conduction parameter. Both hydrodynamically fully developed and developing flow situations were considered. The major finding is that axial conduction along the plate can considerably decrease the energy transferred from the hot fluid to the cold fluid for flows having low Peclet numbers.

Nomenclature

c_p = specific heat, J/kg·°C
C = capacitance flow rate ($=\dot{m}c_p$), W/°C
d = distance from plate surface to fluid centerline, m
k = thermal conductivity, W/m·°C
L = length of heat exchange section, m
$\dot{m}$ = mass flow rate, kg/s
Pe = Peclet number ($=RePr$)
Pr = Prandtl number ($=\nu/\alpha$)
Re = Reynolds number ($=U_0 d/\nu$)
t = half plate thickness, m
T_h = hot fluid inlet temperature, °C
T_c = cold fluid inlet temperature, °C
u = dimensionless x velocity component ($=u'/U_0$)
u' = dimensional x velocity component, m/s
U_0 = inlet velocity, m/s
v = dimensionless y velocity component ($=v'/U_0$)
v' = dimensional y velocity component, m/s
x = dimensionless coordinate direction ($=x'/d$)
x_u = dimensionless unheated length
x' = dimensional coordinate direction, m
y = dimensionless coordinate direction ($=y'/d$)
y' = dimensional coordinate direction, m
α = thermal diffusivity, m^2/s
ϵ = effectiveness
θ = dimensionless temperature
λ = wall conduction parameter [$=(2t/d)(k_s/k_f)$]
τ = temperature, vorticity or stream function
ω = dimensionless fluid vorticity
ψ = dimensionless stream function
ν = kinematic viscosity, m^2/s

Subscripts

b = bulk property of fluid
c = cold fluid
e = exit
f = fluid
h = hot fluid
i,j = ith row, jth column in the x-y plane
s = solid
0 = inlet

Presented as Paper 82-0833 at the AIAA/ASME Third Joint Thermophysics, Fluids, Plasma and Heat Transfer Conference, St. Louis, Mo., June 7-11, 1982; submitted May 1, 1983; revision received Aug. 18, 1983. Copyright © American Institute of Aeronautics and Astronautics, Inc., 1983. All rights reserved.

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Introduction

IN conventional convective heat-transfer problems, the thermal boundary conditions at the solid walls are assumed to be known, either in terms of the temperature or heat flux or a combination of both. The temperature field in the solid is obtained by introducing an assumed heat-transfer coefficient into the boundary conditions. However, in the broader view, the boundary conditions at the interface are not known a priori but depend on the coupled conduction-convection mechanism. Such a problem, in which both of these two interface conditions are treated simultaneously as unknowns, is known as a conjugated heat-transfer problem.

The literature on conjugated problems has not been extensive. Davis and Gill[1] analyzed a conjugated problem between parallel plates with Poiseulle-Couette flow, with one wall at the temperature of the entering fluid and the outside of the other wall at constant-heat flux. They found that the axial wall conduction can significantly affect the fluid temperature field and lower the Nusselt number relative to that predicted for a uniform heat flux at the solid/fluid interface. They found fair agreement between the experimental results and their analysis.

Mori et al.[2] analyzed a conjugated problem for parallel plates that was similar to that of Davis and Gill. For the case of specified constant heat flux at the outside wall, again axial heat conduction along the wall reduces the Nusselt number. The lowest magnitude of the interface Nusselt number corresponds to that with constant temperature at the solid/fluid interface as the boundary condition. For the case of specified constant wall temperature at the outside wall, the finite wall resistance increases the interface Nusselt number. The highest value of the Nusselt number corresponds to that with a constant heat flux at the solid/fluid interface as the boundary condition.

Mori et al.[3] solved the conjugated problem for laminar flow in a circular tube. Like their conjugated problem for

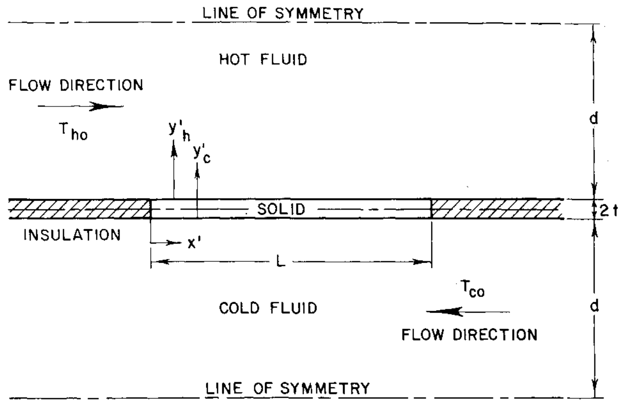

Fig. 1 Model of idealized flat-plate heat exchanger.

laminar flow between parallel plates, they considered the two thermal boundary conditions specified at the outside wall of the tube, namely, constant heat flux and constant temperature. They derived essentially the same conclusions as for parallel plates.

Faghri and Sparrow[4] investigated simultaneous wall and axial fluid conduction in laminar pipe flow. They observed that axial conduction in the tube wall can cause a substantial preheating of both the wall and fluid upstream of the region where active heating begins. At low Peclet numbers, axial fluid conduction also causes preheating. However, axial wall conduction may readily overwhelm axial fluid conduction.

In all of the above investigations, the boundary conditions on the external solid boundary were prescribed by either a known temperature distribution or heat flux. These studies elucidate and quantify the significance of wall conduction on convective heat transfer. The objective of this paper is to investigate the heat-transfer characteristics between two counterflowing fluids on either side of a heat-conducting plate. Unlike the earlier investigations, the boundary conditions at both solid/fluid interfaces are not known. Instead, the boundary conditions are replaced by an energy balance.

Problem Formulation

A description of the problem is shown in Fig. 1. The hot fluid flows along the top of the plate, while the cold fluid flows along the bottom in a countercurrent fashion. Both fluid flows are assumed to be laminar. The plate is divided into three sections. There are insulated sections at both ends, in which, no heat transfer across the plate is allowed. The heat exchange section is in the middle where any heat transfer between the fluids occurs. The insulated sections allow a clear demonstration of the effects of axial fluid conduction. It was shown[5] for Peclet numbers of less than 50 that axial fluid conduction is important. Specifically, the hot fluid can lose energy and the cold fluid can gain energy prior to their entry into the heat exchange section. The plate can be one of a series that might be used to make up the components of an idealized laminar flat-plate heat exchanger. Therefore, the fluid centerlines parallel to the plate may be considered as axes of symmetry. No energy is transferred across these centerline boundaries.

In the first part of this investigation, the hot and cold fluids are assumed to have a fully developed parabolic velocity profile throughout the entire flow length. Later, both fluids are assumed to enter with a slug flow profile, and the hydrodynamic development is also considered. The effects of the flow development on the heat transfer will be analyzed at the end of this paper.

The inlet temperatures of the fluids are assumed to be at a constant value. This can be justified if the insulated regions are sufficiently long. The convective heat-transfer coefficients at the solid/fluid interfaces are not known. Rather, an energy balance is used. The temperature distribution in the plate is assumed to be one-dimensional. The wall temperature is not known a priori. It is determined as a result of the energy balance mentioned earlier. It should be noted here that the assumption of one-dimensional heat conduction along the plate is an acceptable approximation when the thickness of the plate is less than 1% of its length, i.e., $2t/L<0.01$. An attempt to solve for the two-dimensional temperature distribution in the plate was made and the results show that the temperature change across the plate is negligible compared to the temperature change along the plate.

From the above description it is clear that the present problem is a conjugated heat-transfer one, since the temperature fields of the hot fluid, the plate, and the cold fluid must be solved for simultaneously with no prior knowledge of the temperature or heat flux at the solid/fluid interfaces.

Analysis and Method of Solution

Governing Equations

For the cases where the velocity profiles of both the hot and cold fluids are assumed to be fully developed, there is no need to solve the vorticity or velocity equations. However, for the hydrodynamically developing cases, the velocity fields must also be solved with a slug flow profile at the entrances of both the hot and cold fluids. The flow length is assumed to be sufficiently long so that both fluids will be fully developed at the exits.

The following forms were used for the dimensionless variables:

$$x=x'/d, \quad y_h=y'_h/d, \quad y_c=y'_c/d$$

$$u_h=u'_h/U_{h0}, \quad v_h=v'_h/U_{h0}, \quad u_c=u'_c/U_{c0}$$

$$v_c=v'_c/U_{c0}, \quad \omega=\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}, \quad \theta_h=\frac{2(T_h-T_{\text{ave}})}{T_{h0}-T_{c0}}$$

$$\theta_c=\frac{2(T_c-T_{\text{ave}})}{T_{h0}-T_{c0}}, \quad \theta_s=\frac{2(T_s-T_{\text{ave}})}{T_{h0}-T_{c0}} \tag{1}$$

where $T_{\text{ave}}=(T_{h0}+T_{c0})/2$.

In dimensionless form, the vorticity, stream function and x, y velocity components satisfy the following equations:

$$u\frac{\partial\omega}{\partial x}+v\frac{\partial\omega}{\partial y}=\frac{1}{Re}\left[\frac{\partial^2\omega}{\partial x^2}+\frac{\partial^2\omega}{\partial y^2}\right] \tag{2}$$

$$\omega=\nabla^2\psi \tag{3}$$

$$u=\frac{\partial\psi}{\partial y} \tag{4}$$

$$v=-\frac{\partial\psi}{\partial x} \tag{5}$$

These equations were applied only to the developing flow cases.

The hydrodynamic boundary conditions for the developing case may be summarized as follows:

1) At the plate surfaces

$$-x_u\leq x\leq L/d+x_u, \quad y_h=0; \quad \omega_h=\frac{\partial^2\psi_h}{\partial y_h^2}, \quad \psi_h=0$$

$$y_c=0; \quad \omega_c=\frac{\partial^2\psi_c}{\partial y_c^2}, \quad \psi_c=0$$

2) At the fluid centerlines

$$-x_u \leq x \leq L/d + x_u, \quad y_h = 1; \quad \omega_h = 0, \quad \psi_h = 1$$
$$y_c = -1; \quad \omega_c = 0, \quad \psi_c = 1$$

3) At the fluid entrances

$$x = -x_u, \quad 0 < y_h < 1; \quad \omega_h = 0, \quad \psi_h = y_h$$
$$x = L/d + x_u, \quad -1 < y_c < 0; \quad \omega_c = 0, \quad \psi_c = -y_c$$

4) At the fluid exits

$$x = L/d + x_u, \quad 0 < y_h < 1; \quad \omega_h = 3(1 - y_h)$$
$$\psi_h = -\frac{3}{2}(y_h^3/3 - y_h^2)$$
$$x = -x_u, \quad -1 < y_c < 0; \quad \omega_c = 3(1 + y_c)$$
$$\psi_c = \frac{3}{2}(y_c^3/3 + y_c^2)$$

For the fully developed flow situations, a parabolic velocity profile was assumed throughout the flow length. Therefore, it was not necessary to solve for the vorticity and stream function in these situations. For fully developed flow throughout the flow length, the velocity distributions were prescribed as follows:

$$-x_u \leq x \leq L/d + x_u, \quad 0 < y_h < 1; \quad u_h = \frac{3}{2}(-y_h^2 + 2y_h)$$
$$v_h = 0$$
$$-1 < y_c < 0; \quad u_c = \frac{3}{2}(y_c^2 + 2y_c)$$
$$v_c = 0$$

The conjugated heat-transfer problem, which is considered here, is governed by the energy equation for the hot and cold fluids and by an energy balance for the plate. After being written in dimensionless variables, the energy equation for the fields becomes

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pe}\left[\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right] \tag{6}$$

For the solid, an energy balance was performed. A one-dimensional temperature distribution is adequate for small values of t/L. If it may be assumed that the thermal conductivities of both the hot and cold fluids are equal, the resulting equation may be written in the followng form:

$$\frac{\partial \theta_c}{\partial y_c} - (2t/d)(k_s/k_f)\frac{\partial^2 \theta_s}{\partial x^2} - \frac{\partial \theta_h}{\partial y_h} = 0 \tag{7}$$

From Eq. (7), it is clear that $\lambda[=(2t/d)(k_s/k_f)]$ is an important parameter relating to the effects of axial wall conduction on the heat transfer between the hot and cold fluids. The thermal boundary conditions may be stated as follows:

1) At the fluid centerlines

$$-x_u \leq x \leq L/d + x_u, \quad y_h = 1; \quad \frac{\partial \theta_h}{\partial y_h} = 0$$
$$y_c = -1; \quad \frac{\partial \theta_c}{\partial y_c} = 0$$

2) At the fluid entrances

$$x = -x_u, \quad 0 < y_h < 1; \; \theta_h = 1$$
$$x = L/d + x_u, \quad -1 < y_c < 0; \; \theta_c = -1$$

3) At the fluid exits

$$x = L/d + x_u, \quad 0 < y_h < 1; \quad \frac{\partial \theta_h}{\partial x} = 0$$
$$x = -x_u, \quad -1 < y_c < 0; \quad \frac{\partial \theta_c}{\partial x} = 0$$

4) At both ends of the plate

$$x = 0 \text{ and } x = L/d; \quad \frac{\partial \theta_s}{\partial x} = 0$$

5) Along unheated regions of the plate

$$-x_u \leq x \leq 0 \text{ and } L/d \leq x \leq L/d + x_u, \quad y_h = 0; \quad \frac{\partial \theta_h}{\partial y_h} = 0$$
$$y_c = 0; \quad \frac{\partial \theta_c}{\partial y_c} = 0$$

For the heat-transfer region of the plate, Eq. (7) is used when axial conduction effects are included. If axial wall conduction is not taken into consideration, the following relation is used:

$$\frac{\partial \theta_h}{\partial y_h} = \frac{\partial \theta_c}{\partial y_c} \tag{8}$$

Solution Methodology

The governing differential equations were set up in finite difference forms using the exponential difference scheme.[6] This scheme was shown to be very accurate with a moderate number of grid points when the fluid flow is predominantly one-dimensional.[7,8] If the velocity development was needed, the vorticity and stream function equations were solved first by an iteration procedure with under-relaxation for the vorticity equation. The vorticity at the plate was determined by a second-order one-sided approximation for $\partial^2\psi/\partial y^2$. This was given by[9]

$$\omega_w = [-7\psi_w + 8\psi_{w+1} - \psi_{w+2}]/2\Delta y^2 \tag{9}$$

where the subscripts w, $(w+1)$, and $(w+2)$ denote the wall grid point and the first and second grid points from the wall, respectively. The iterative procedure stopped when the maximum difference between the $k-1$ and kth iterations was less than the convergence criterion to be given later.

If the flows at the entrances were already prescribed as fully developed, then the above calculations were not carried out, but the parabolic velocity profiles were used.

After the hydrodynamic solution was known, the temperature fields for both the hot and cold fluids as well as for the solid were solved simultaneously. Along the heat exchange region of the plate, the energy balance given by Eq. (7) was put into a finite difference formulation. Since the transverse thermal resistance of the plate was neglected,

$$\theta_h = \theta_s = \theta_c \tag{10}$$

the temperature at a grid point in the solid becomes

$$\theta_{i,w} = [(\theta_{ci,w-1} + \theta_{hi,w+1})B + \theta_{si+1} + \theta_{si-1}]/2(B+1) \tag{11}$$

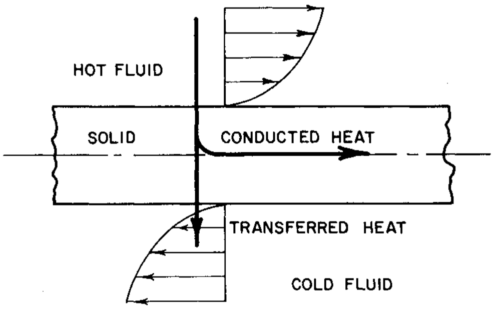

Fig. 2 Paths of heat flow.

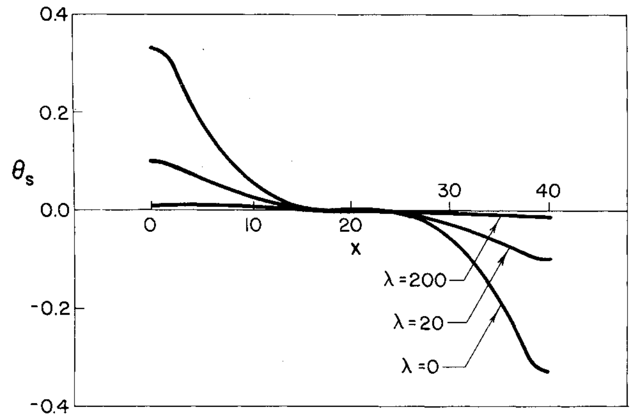

Fig. 3 Plate temperature distribution for $Pe = 1$.

where $B = \Delta x^2 / (\lambda \Delta y)$ and $(w-1)$ and $(w+1)$ denote the grid points just below and above the solid, respectively.

For the limiting case where axial conduction along the plate is ignored, the following equation is used:

$$\theta_{i,w} = (\theta_{hi,w+1} + \theta_{ci,w-1})/2 \tag{12}$$

The iteration procedure for the temperature distribution began by evaluating the exit and the centerline temperatures for the hot fluid. Then the temperature field within the hot fluid was evaluated. The temperature distribution along the plate was then evaluated using the equation for the heat-conducting region. Following this, the cold fluid temperature field was evaluated. Then, the cold fluid centerline and exit temperatures were calculated. Similar to the hydrodynamic solution the maximum difference between the $k-1$ and kth iteration was checked to see if the convergence criterion had been satisfied. If not, the procedure began once again by evaluating the hot fluid boundaries and proceeding as described above. Once again, successive under-relaxation was used in the iterative procedure.

The convergence criterion used for the stream function, vorticity, and temperature fields was

$$|\tau_{i,j}^k - \tau_{i,j}^{k-1}|_{\max} / \tau_{\max} < 10^{-4} \tag{13}$$

where $\tau_{i,j}$ represents either the steam function, vorticity, or temperature and k the iteration counter.

Upon obtaining the converged temperature fields, the bulk temperatures for both the hot and cold fluids were calculated from the following equation:

$$\theta_b = \frac{1}{\bar{u}} \int_0^1 \theta u \, \mathrm{d}y \tag{14}$$

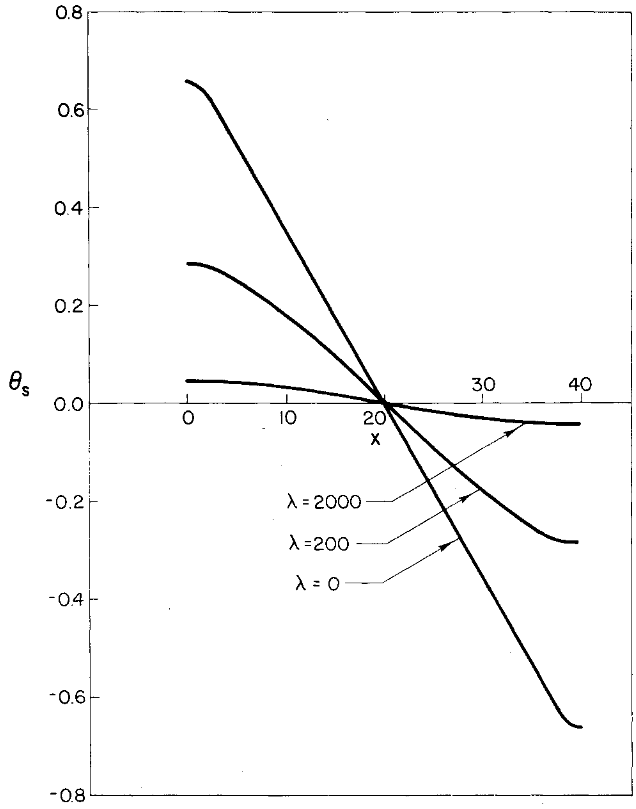

Fig. 4 Plate temperature distribution for $Pe = 10$.

where $\bar{u} = 1$ for the hot fluid and $\bar{u} = -1$ for the cold fluid.

The thermal performance of the heat exchange between the hot and cold fluids was evaluated by using the standard definition of effectiveness for a heat exchanger.

The effectiveness of a heat exchanger may be defined by the following relations depending on its capacity flow rate ratio:

$$\epsilon = \frac{C_h}{C_{\min}} \left[\frac{T_{h0} - T_{he}}{T_{h0} - T_{c0}} \right] = \frac{C_c}{C_{\min}} \left[\frac{T_{ce} - T_{c0}}{T_{h0} - T_{c0}} \right] \tag{15}$$

Upon substituting the dimensionless variables, the relations become

$$\text{for } C_h \le C_c, \quad \epsilon = (1 - \theta_{bhe})/2$$

$$\text{for } C_c \le C_h, \quad \epsilon = (1 + \theta_{bce})/2 \tag{16}$$

The values presented in the next section were found using an Amdahl 470-V7 high-speed digital computer. In the x direction 57 grid points were used and in the y direction 17. By employing a finer grid on a few cases, the accuracy of the results on the plate and bulk temperatures in the next section is judged to within 2%. The accuracy of the velocity and temperature fields is within 5%. No efforts to minimize the CPU time were taken. A typical computer run took about 5-10 min of CPU time.

Results and Discussion

A significant effect on heat transfer between the hot and cold fluids due to axial conduction was observed. The mechanism involved may best be understood upon examining Fig. 2. Illustrated here are the paths along which energy may flow upon leaving the hot fluid. For laminar flow, the heat conducted along the plate may be of the same magnitude as the energy transferred to the cold fluid. An important parameter, $\lambda = (2t/d)(k_s/k_f)$, which indicates the significance of axial wall conduction, is evident from Eq. (7). For a certain amount of energy from the hot fluid, this wall

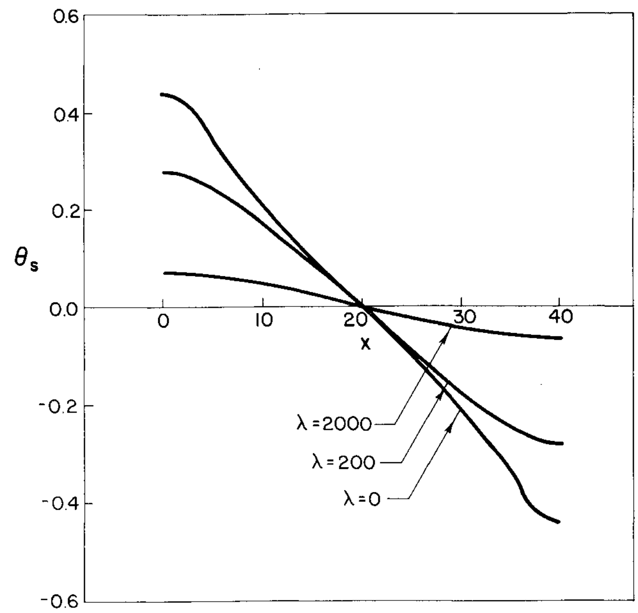

Fig. 5 Plate temperature distribution for $Pe = 50$.

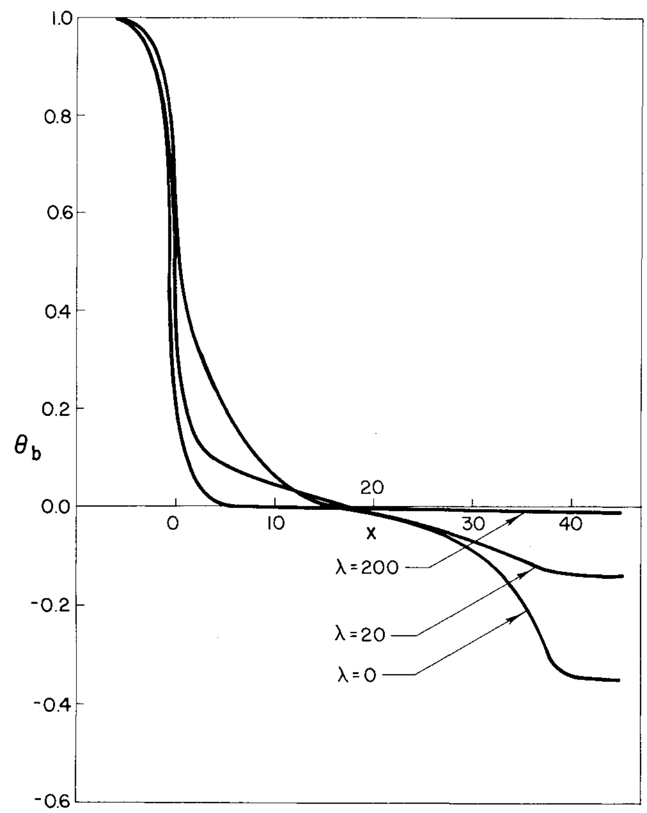

Fig. 6 Hot fluid bulk temperature distribution for $Pe = 1$.

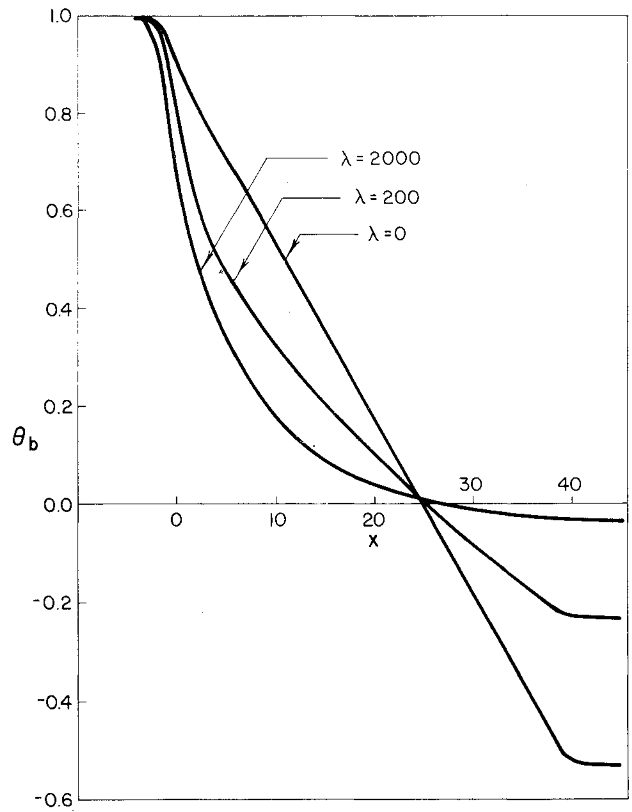

Fig. 7 Hot fluid bulk temperature distribution for $Pe = 10$.

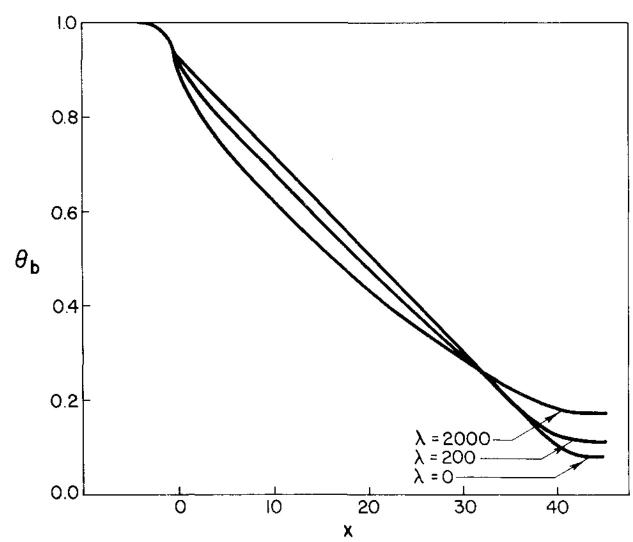

Fig. 8 Hot fluid bulk temperature distribution for $Pe = 50$.

conduction parameter determines the relative amount of energy to be conducted along the plate compared to the amount going into the cold fluid.

The energy equations of the hot and cold fluids indicate that the Peclet number is also an important parameter. Focusing on the hot fluid, if the Peclet number is large, the fluid conduction is small compared with axial bulk convective transport. This would imply that the energy farther away from the plate will be convected downstream and little will be transported to the plate. Hence, the bulk temperature will drop very slowly and little energy will be transferred to the cold fluid. In this situation, the effect of axial wall conduction is not expected to be significant.

However, for low Peclet numbers, heat transfer across the hot fluid to the plate will be much higher. The temperature of the plate determines how much energy is transferred from the hot fluid to the plate. The wall conduction parameter determines, in part, how much of the energy from the hot fluid will be conducted axially along the plate. Since the effects of axial wall conduction are more significant for low Peclet number flows, three low Peclet numbers are chosen in the present study, namely, 1, 10, and 50.

For hydrodynamically developing flow, the Reynolds number is also an important parameter as it will determine the development of the velocity profile. For fully developed flow,

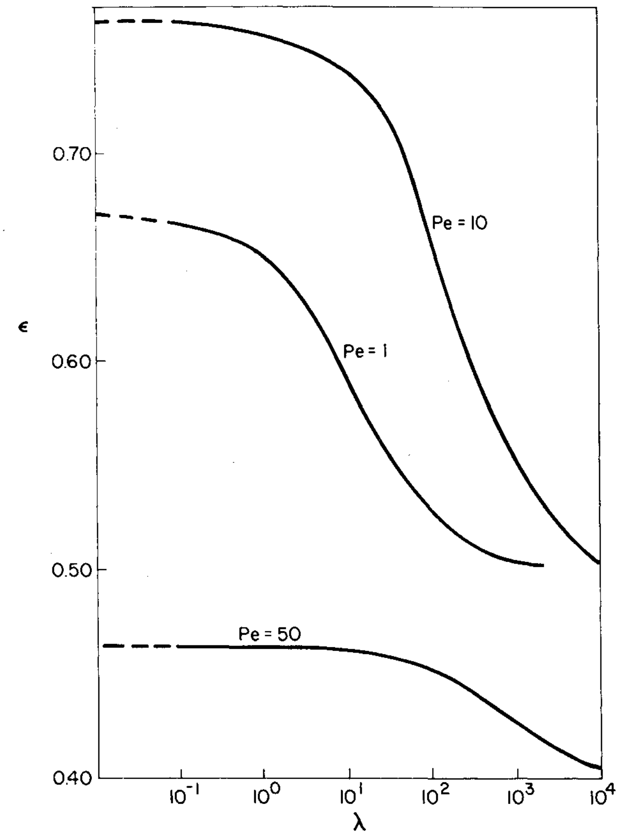

Fig. 9 Effects of axial wall heat conduction on the thermal performance.

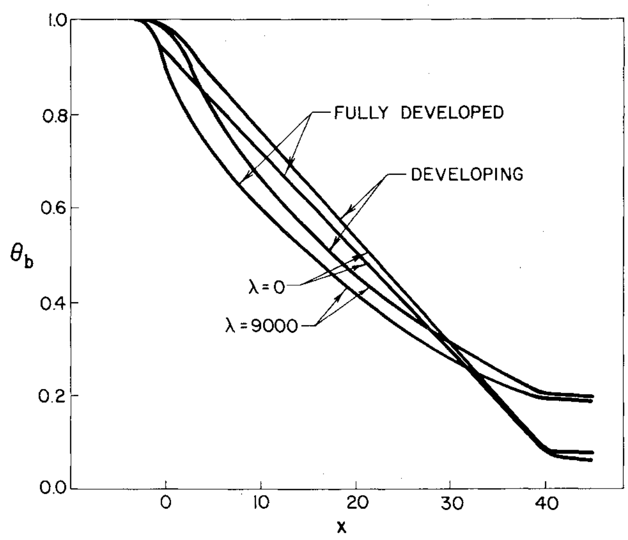

Fig. 10 Effects of flow development on the hot fluid bulk temperature distribution for $Re = 50$ and $Pe = 50$.

the velocity profile is always parabolic along the entire flow path. In this case, only the Peclet number is relevant.

In this paper, balanced flow is assumed, that is $C_h = C_c$. In addition, the hot and cold fluids are assumed to have the same physical and thermal properties. The dimensionless heat exchange section length is 40 and the insulated sections at both ends have a length of 8. This insulated length is long enough so that the entry temperature profiles for both hot and cold fluids can be assumed to be constant. It can be seen from the results given later in this paper that for $Pe \geq 1$, the axial conduction cannot affect the temperature fields beyond a dimensionless length of four or five prior to entering or after leaving the heat exchange section. Hence, the boundary conditions stated in the previous section are valid.

Since the temperature distributions of the hot and cold fluids are antisymmetric, only the results for the hot fluid will be presented.

Plate Temperature

Figures 3-5 show the temperature along the plate for Peclet numbers of 1, 10, and 50, respectively. The velocity profiles for both hot and cold fluids are fully developed at the entrances. For $Pe = 1$, Fig. 3 shows that the axial plate temperature distribution is very sensitive to the wall conduction parameter. For $\lambda = 200$, the plate temperature is essentially zero, indicating extremely effective heat conduction along the plate. For an infinitely conducting plate, the plate temperature should be zero.

Figures 4 and 5 show essentially the same trend for the dependence of plate temperature on the wall conduction parameter for $Pe = 10$ and 50, but to a lesser extent. For $Pe = 10$, the plate temperature is nearly uniform when the conduction parameter is greater than 2000, whereas for $Pe = 50$, the plate temperature is almost uniform when the wall conduction parameter is greater than 6000.

It should be noted that the value of θ_s at $x = 0$ is not monotonic with Pe. For example, with $\lambda = 0$, $\theta_s(0)$ increases from a value of about 0.33 at $Pe = 1$ to a value of about 0.67 at $Pe = 10$ and then decreases to a value of about 0.43 at $Pe = 50$. The explanation for this behavior can be explained by investigating the limits of $\theta_s(0)$ for very small and large values of Pe. As $Pe \to 0$, fluid conduction dominates the entire heat-transfer process. This means the temperature fields of the hot and cold fluids as well as the plate are all approximately the same. This yields a value of $\theta_s(0)$ close to zero. As $Pe \to \infty$, axial bulk convective transport dominates an essentially no energy is transferred between the hot and cold fluids. In this case, the plate temperature is again nearly equal to zero.

Bulk Temperature

The bulk temperature of the hot fluid is plotted as a function of axial location in Figs. 6-8 for different values of the wall conduction parameter for $Pe = 1$, 10, and 50. The velocity profiles are fully developed everywhere. Again, since the temperature distribution for the hot and cold fluids are antisymmetric, only the results for the hot fluid need to be presented.

For $Pe = 1$, the axial fluid conduction effects are very important as is seen in Fig. 6. A drop in the bulk temperature upstream of the heat exchange region is evident. Before the hot fluid reaches the heat exchange section, depending on the wall conduction parameter, the bulk temperature can drop to 0.6 or less. This precooling of the hot fluid would not occur if the axial fluid conduction term is ignored in Eq. (6). The $\lambda = 0$ curve denotes the bulk temperature distribution when the axial wall conduction effects are ignored. As the wall conduction parameter λ increases, the bulk temperature distribution becomes flatter and levels off to a higher value at the exit. The exit bulk temperature is the lowest when $\lambda = 0$. For $\lambda = 200$, the bulk temperature approaches zero at a value of x of about six and remains nearly constant throughout the remaining part of the plate. It is evident that increasing λ above 200 would not change the bulk temperature significantly because for such λ, the bulk temperature of the hot fluid cannot drop to below the plate temperature, which is nearly zero along the whole plate.

The results at $Pe = 10$ are shown in Fig. 7. The precooling of the hot fluid is not as severe as compared to at $Pe = 1$. Still, as much as 20% of the energy of the hot fluid may be lost before reaching the heat exchange section. As the conduction parameter increases, the bulk temperature initially falls rapidly but levels off to a higher value compared to $\lambda = 0$. For

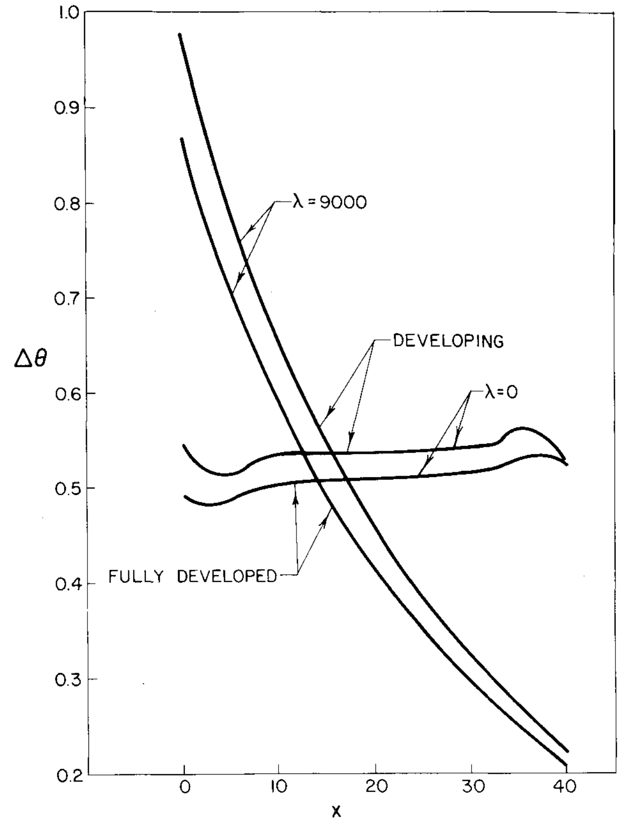

Fig. 11 Effects of flow development and axial heat conduction on the difference between the hot fluid bulk temperature and the solid temperature for $Re = 50$ and $Pe = 50$.

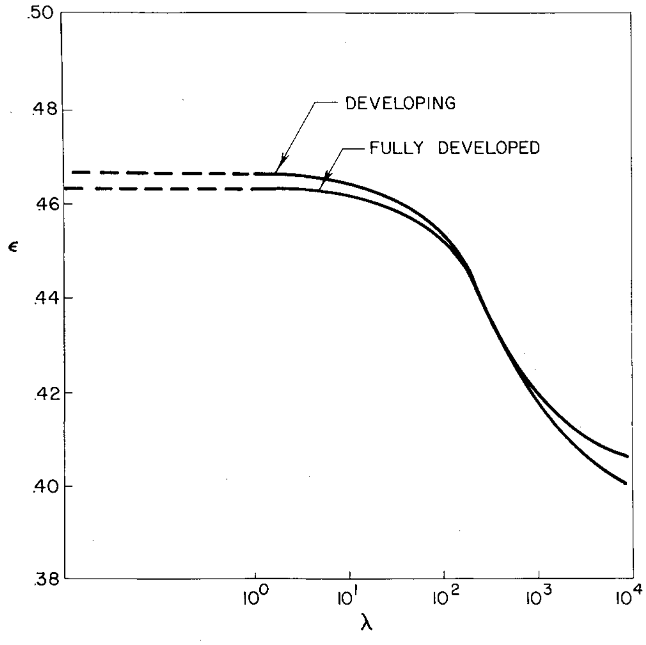

Fig. 12 Effects of flow development and axial heat conduction on the thermal performance for $Re = 50$ and $Pe = 50$.

values of the conduction parameter of 2000, the bulk temperature at the exit approaches the limiting value of zero.

It should be noted that for both of the previous cases, the exit bulk temperature is less than zero for the case $\lambda = 0$. As the wall conduction parameter increases, the exit bulk temperature approaches zero. However, as Fig. 8 shows, at $Pe = 50$, the exit bulk temperature for $\lambda = 0$ is higher than zero and the exit bulk temperature increases further as the wall conduction parameter increases. Also, as shown in Fig. 8, the dependence of the bulk temperature distribution on the wall conduction parameter λ follows the same trend as in the previous two cases, but the dependence on λ is to a lesser degree. As expected, for $Pe = 50$, the precooling of the hot fluid is little compared with those for $Pe = 1$ and 10. This is consistent with the results of previous investigators.[4,5]

Thermal Performance

Considering the plate as one of many in an idealized laminar flat-plate heat exchanger, the total heat transfer between the hot and cold fluid can be represented by the thermal effectiveness defined earlier. In Fig. 9, the effectiveness is plotted as a function of the dimensionless wall conduction parameter λ for the three aforementioned Peclet numbers. The points on the vertical axis (which should actually be at minus infinity to the left) represent the effectiveness for $\lambda = 0$. In the figure, these points are connected to the rest of the curves by a dashed line. It is evident that the effectiveness begins to decrease as soon as the value of λ is larger than zero. For a Peclet number of 1, a rapid decline in the effectiveness occurs when the value of the wall conduction parameter lies between 1 and 200. For values greater than 200, the effectiveness begins to level off to 0.5.

For a Peclet number of 10, the range of values of λ for this steep sloping portion is between 20 and 2000. Above 2000, the effectiveness again levels off to 0.5.

The effectiveness deteriorates from 0.67 for $\lambda = 0$ to 0.5 for large λ for the case of $Pe = 1$. For $Pe = 10$, the deterioration is from 0.76 to 0.5. For a Peclet number of 50, the thermal effectiveness ϵ deteriorates from 0.46 to about 0.41 as λ is increased from 0 to 9000. For $Pe = 50$, most of the drop in the effectiveness occurs in the range of the wall conduction parameter between 20 and 5000.

An interesting point that should be noted is that the effectiveness ϵ is not monotonic with respect to Pe. Specifically, the effectiveness is higher at $Pe = 10$ than at $Pe = 1$. The explanation for this is similar to that given earlier for the nonmonotonic behavior of the plate temperature with Pe. As $Pe \to 0$, since the temperatures of both the hot and cold fluids are nearly the same (nearly equal to zero), the effectiveness should be about 0.5. As $Pe \to \infty$, the hot and cold fluids will have a temperature of about 1 and -1 throughout, respectively. In this situation, the effectiveness is close to zero.

Developing Flow vs Fully Developed Flow

In Figs. 10-12, comparisons are made between the hydrodynamically fully developed and developing flows. The Reynolds and Peclet numbers are both 50. It can be shown that at this Reynolds number, the flow becomes fully developed for $x \geq 2$. Therefore, the convective heat-transfer coefficients are about the same along most of the flow length for both situations under consideration.

Figure 10 illustrates the effects of flow development on the bulk temperature distribution of the hot fluid. One observation to be made is that the precooling effect for fully developed flow is greater than for developing flow. This is due to a lower velocity near the solid surface for x less than two. Because of this precooling effect, the fully developed flow starts out at a lower bulk temperature and stays lower throughout most of the flow length. For low values of the wall conduction parameter, there is a crossing of the bulk temperature distributions near the exit of the heat exchange region. Whether the profiles cross or not is dependent on the amount of heat transfer that takes place in the channel between the hot fluid and the plate. This is related to the convective heat-transfer coefficients, which have been shown to be equal throughout most of the channel, and the temperature difference between the hot fluid and the plate.

The temperature difference between the hot fluid and the plate is shown in Fig. 11 for the wall conduction parameter of 0 and 9000. One observation that can be made is, for both values of the wall conduction parameter, the temperature difference for the developing flow situation is consistently higher. Therefore, the heat transfer for the developing flow is always higher. For a small wall conduction parameter, most of the energy from the hot fluid is transferred to the cold fluid. In this instance, the temperature difference remains high and large amounts of energy will be transferred throughout the entire flow length. Since the heat transfer remains high for x greater than midchannel and the heat transfer for developing flow is always higher, this allows the developing flow bulk temperature to drop to a value below that of the fully developed flow near the exit.

When the wall conduction parameter is large, the heat transfer between the hot fluid and the plate for x less than midchannel is high owing to the high temperature difference. A significant amount of this energy, however, is conducted axially along the plate. For x greater than midchannel, the plate temperature is sufficiently high so as to bring the temperature difference between the hot fluid and the plate to a low value, thereby causing the heat transfer to be low. Even though the heat transfer between the hot fluid and the plate is higher for developing flow, the excess heat transfer is insufficient to allow the bulk temperature distributions for developing and fully developed flows to cross at any point.

In Fig. 12, the effectiveness is shown for values of the wall conduction parameter of 0-9000 for both developing and fully developed flows. It is shown in the figure that for the wall conduction parameter below 200, the developing flow has higher effectiveness. Above this value, the fully developed flow is more effective.

Summary and Conclusions

The effects of the heat-conducting plate separating two counterflowing fluids are to lower the bulk temperature of the hot fluid in the early region and to keep the bulk temperature at a higher value near the exit. Considering this plate as one of a series of thin heat-conducting plates that make up an idealized laminar flow flat-plate heat exhanger, the effects of increasing axial wall conduction are to decrease the effectiveness of the plate.

The axial wall conduction effects were found to depend on a wall conduction parameter that is the product of the dimensionless plate thickness and the thermal conductivity ratio between the solid and fluid. It was observed that axial wall conduction effects started to become significant at a lower value of the wall conduction parameter for lower Peclet numbers. This paper considers the situations of balanced flows only and only when the hot and cold fluids have the same physical and thermal properties. For a Peclet number of 1, a steep drop in the effectiveness occurred when the wall conduction parameter reached about 1. For Peclet numbers of 10 and 50, a similar occurrence did not start until the wall conduction parameter was over 20 and 200, respectively.

A comparison was made on heat transfer between developing flow and fully developed flow. The effects of axial fluid conduction are more significant for fully developed flow due to the lower fluid velocities near the plate. For low values of the wall conduction parameter, the heat transfer between the hot and cold fluids are higher for developing flow, whereas for high values of the wall conduction parameters the fully developed flow case yields higher heat transfer between the two fluids.

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